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Inverse Distance Weighting for Extrapolating Balloon-Directivity-Plots

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ABSTRACT

This paper investigates an extrapolation for missing directivity-data in connection with Balloon-Plots. Such plots display the spherical directivity-pattern of radiators and receivers in form of contoured sound pressure levels. Normally the directivity-data are distributed evenly so that at each point of the display-sphere there would be sufficient data-points. However, there are circumstances where we want to display data that are not evenly distributed. For example, there might be only available the horizontal and vertical scans. The proposed Inverse Distance Weighting method is a means to extrapolate into these gaps. This paper explains this method and demonstrates some examples.

1. INTRODUCTION

The radiation characteristic of electro-acoustic devices can be measured or simulated by sampling over a sphere of a given radius around the device under test.

Usually the sampling is carried out in spherical coordinates on a regular angular grid, as described in [1]. Once data are collected, the data-set is graphically represented with the help of so called balloon-plots. These plots are in the form of contour plots over a spherical surface or as surfaces in spherical coordinates where the radius is proportional to the gain of the device along the pointed direction.

It may happen that, due to the complexity of measurement infrastructure or non-regular sampling over the spherical surfaces, that there is need to extrapolate data from a reduced or incomplete set of measurement points. On the sphere, therefore, there might be gaps large enough that we do not know the values, hence the usage of the term "extrapolate" in

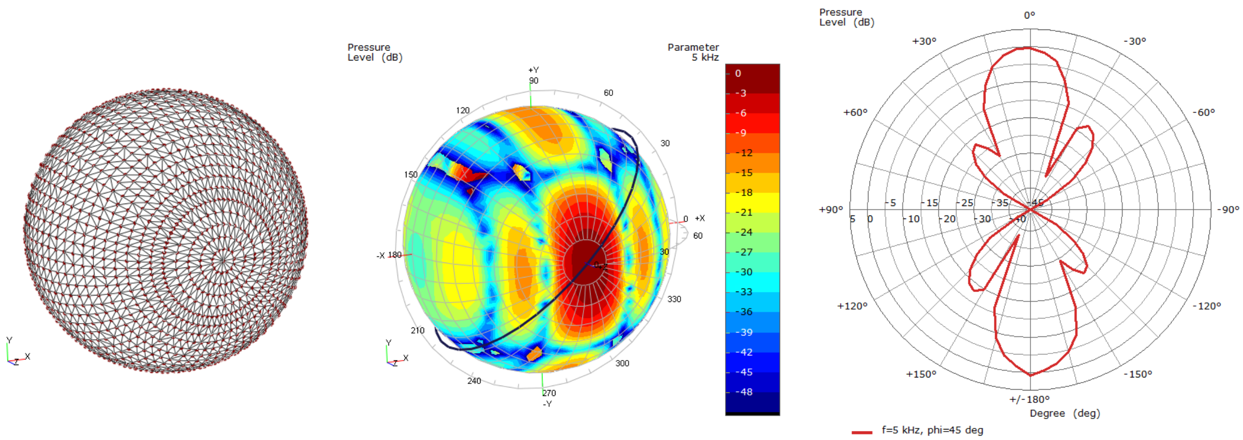

Fig. 1: Test-function at 5 kHz mapped on a mesh of regular polar-scan-lines with an angular resolution of 5 deg. To the right is shown the projection of the levels along the ring at 45 deg onto a polar-plot.

the sense of [2]:

> "In mathematics, extrapolation is the process of constructing new data points. It is similar to the process of interpolation, which constructs new points between known points, but the results of extrapolations are often less meaningful, and are subject to greater uncertainty. It may also mean extension of a method, assuming similar methods will be applicable. ..."

Beside the gaps, dense data might be available. For example a common case is the extrapolation from measurements done on horizontal and vertical scan-lines only.

Because on the sphere any point is naturally "surrounded" by other points, we could perform any of the interpolation-schemes in order to obtain an intermediate value [3][4]. However, in this paper, we would like to draw attention to a particular scheme, which is straightforward to apply and which can be used for interpolation as well as for extrapolation problems. It seems to cope well also with anisotropic distribution of points. The approach is based on [5]. We have adapted it for spherical coordinates and we call it *Inverse Distance Weighting* (IDW) as described in section 2.1.2.

In the first part of this paper we investigate the extrapolation applied to a canonical test-function on an anisotropic grid. For comparisons two approaches are shown: 1) Decomposition with Spherical Harmonics. 2) The Inverse Distance Weighting. The first part also explains the Inverse Distance Weighting.

The second part demonstrates the application of the Inverse Distance Weighting method to a set of measurement curves.

2. TEST FUNCTION

The "SincSinc"-function features the directivity of a rectangular piston in an infinite baffle [6].

$$D\left(k_x, k_y\right) = sinc\left(\frac{L_x}{2}\cdot k_x\right)\cdot sinc\left(\frac{L_y}{2}\cdot k_y\right)$$

with $k_x = k\cdot\cos\left(\varphi\right)\cdot\sin\left(\theta\right)$, $k_x = k\cdot\sin\left(\varphi\right)\cdot\sin\left(\theta\right)$, $k = \omega/c$, ω is angular frequency, c is speed of sound, φ is the azimuthal and θ is polar angle in spherical coordinates. L_x and L_y are the dimensions of the rectangle.

For all examples the test-function is used with following values: L_x=0.2 m, L_y=0.1 m, c=343 m/s. A frequency f=5 kHz is used.

The highest angular resolution of any balloon-mesh is 5 deg. All contours display a range of 50 dB in steps of 3 dB.

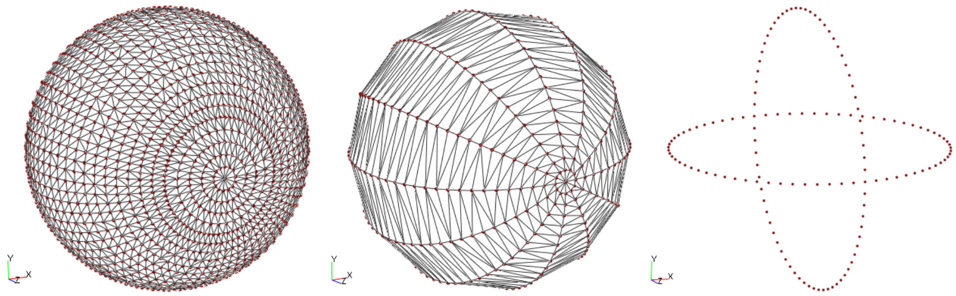

Fig. 2: Spherical grids.

The SincSinc-function turns out to be a good test-function for checking extrapolation schemes for radiation problems, because it features typical radiation patterns including marked interferences (figure 1).

Note, the data-set has been rotated by 2.5 deg about the x-, y- and z-axis in order to provide a simple regularization for following analysis. With the help of the slight shift we make sure that target points do not coincide with source points and thus true interpolation is always performed.

2.1. Extrapolation Example

In the following we want to demonstrate the basic problem by constructing an example, which would makes necessary extrapolation. The idea is to use the SincSinc-function as an example and to calculate values on a non regular grid on the sphere.

An anisotropic grid could be constructed in the following way. To the left of figure 2 we have the regular grid with 5 deg angular resolution. The mesh in the center is anisotropic and provides points at 5 deg resolution in polar direction and 30 deg resolution in azimuthal direction, hence there are wide gaps circumferential. The right plot, finally, pictures the extreme case, where there is a dense distribution of points on vertical and horizontal scan-lines but naught in between.

The idea is to investigate how two selected interpolation schemes cope with the rarefied distributions. Because we know how the function should appear, if sampled properly, we can compare the pictures of the balloon plot and a selected mapping of a polar-plot (along the dark arc) to the original plots as shown in figure 1.

2.1.1. Spherical Harmonics

Intuitively a promising interpolation scheme should be holistic, which means it would take into account all available points in order to magically fill the gaps. Is there a function, which by nature behaves like a "radiator"? If we take many of such functions and weight each of these in such a way that the sum-total satisfies known-values on the sphere, then it should also give reasonable values for the unknown territory. These functions are called Spherical Harmonics or Multipole Expansion [6].

Spherical Harmonics is the natural mode-set of a sphere. In principal any regular distribution of values on a sphere can be assembled from a spectrum of Spherical Harmonics, similar to Fourier and polynomial interpolation in Cartesian coordinates.

The text-book implementation of modal decomposition in Spherical Harmonics works fine as long as each mode gets sufficient data-points to adjust itself. This is one of the reasons why we have added a little rotation to the original data-set, in order to avoid nodal lines of the modes to fall on peculiar scan-lines of the data-set.

For 5/5 deg: The first row of the picture-table 3 is a decomposition with $20{\cdot}20$-modes on a regular 5deg mesh. The decomposition compares well to the original.

For 5/30 deg: The second row shows the decomposition of an anisotropic grid. The algorithm turned out to be stable only up to $6{\cdot}6$ modes. Compared to the original, the balloon-contours and the polar-curve are quite different. However, for a "reach into the unknown" the result may be considered not too bad. First, the overall pattern is reproduced in level.

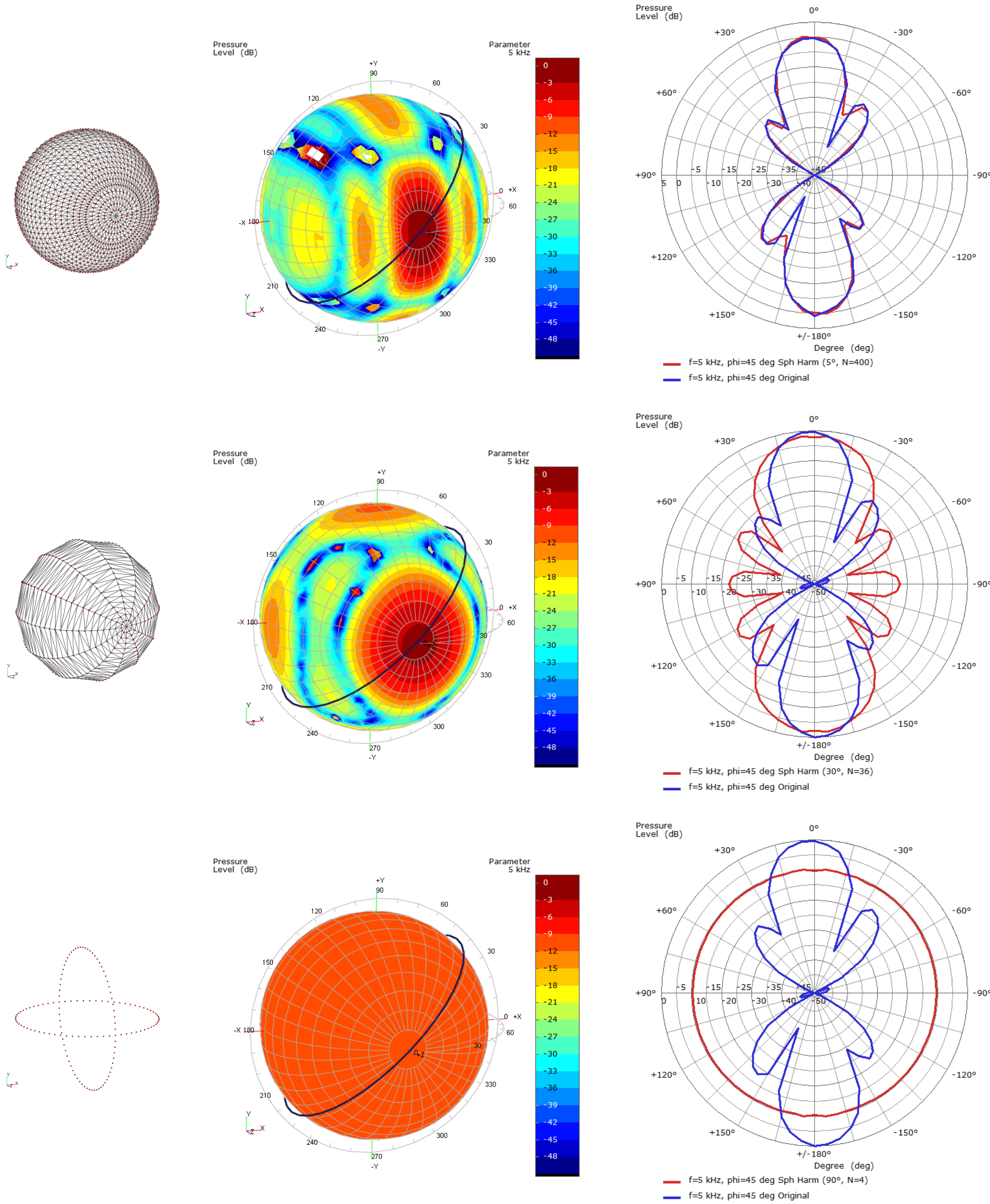

Fig. 3: Extrapolation with Spherical Harmonics.

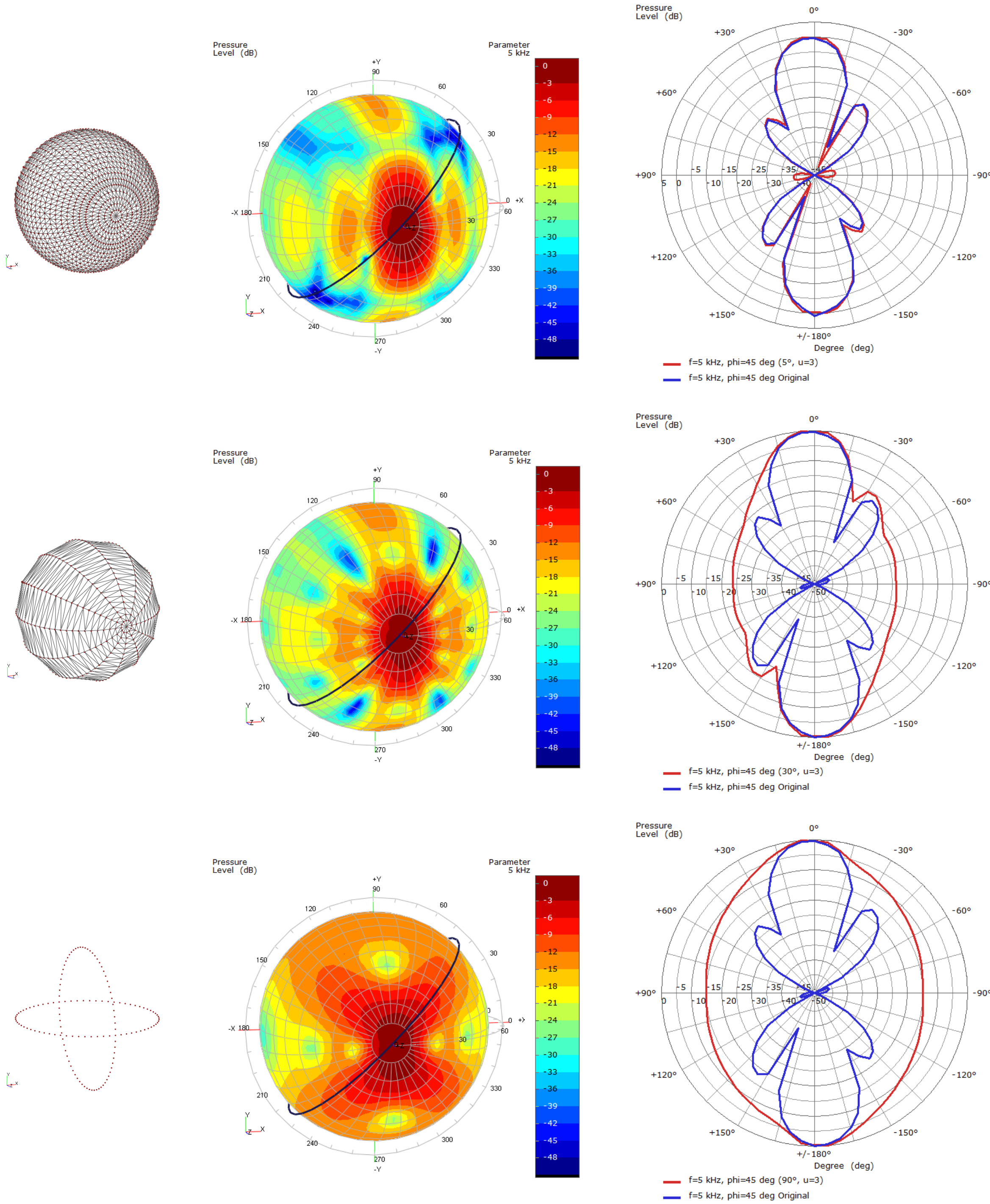

Fig. 4: Extrapolation with IDW.

Second, the Spherical Harmonics yields typical interference patterns of a radiator, which other general extrapolation schemes may not be able to produce.

For 5/90 deg: The modal decomposition algorithm turned out to be unstable for any mode beyond 2·2 (dipoles). However, the result is an omni-directional mean-value.

2.1.2. Inverse Distance Weighting

The Inverse Distance Weighting method (IDW) is a sort of mean-value forming approach for calculating a value at any point on the sphere by taking into account the contribution of all original points.

$$f(P) = \frac{\sum_i d_i^{-u} \cdot z_i}{\sum_i d_i^{-u}}$$

$$f(P) = z_i \,|_{d_i \to 0}$$

with $f(P)$ a value anywhere on the sphere at point P. z_i is a value from the original data-set at point D_i. The function d_i is the surface-distance on the sphere between vectors P and D_i:

$$d_i = \arccos(P \cdot D_i)$$

For Inverse Distance Weighting the factor $u \geq 1$. Typically $u = 2...5$. For the above examples $u = 3$ is used. For interesting details see [5]. If $d_i = 0$ then we substitute the original value z_i at point D_i. It turned out that the algorithm works better if only points from a hemisphere are used with:

$$P \cdot D_i > 0$$

Thus, each data-point contributes according to the inverse of its distance. Data-points, which are close to P provide a strong influence, and data-points further away contribute less. Naturally, the IDW provides a very stable extrapolation as long as $u \geq 1$. However, as the weight-function does not provide oscillations there are no additional interferences produced, which in turn yields the resulting values of the extrapolation often to be too optimistic. Or, in other words, the values further away from the original data-set turn out to be too high compared to the regular sampled version.

Figure 4 provides levels as contours on the balloon surface and polar-plots of the Inverse Distance Weighting method. As in the previous example the original data-set is the SincSinc-function (figure 1) rotated by 2.5 deg about x-, y- and z-axis. The rotation is applied in order to trigger the IDW for all cases. All extrapolations are performed with $u = 3$.

For 5/5 deg: The first row shows the application of the IDW on the regular grid. Because of the rotation by 2.5 deg the IDW provides interpolated values. The results compare well to the pictures of the original function. Some spatial smoothing is present.

For 5/30 deg: The second row shows the result of the IDW for original points distributed on an anisotropic grid. The pattern of the contours is different compared to the original balloon, however, the main features are maintained. The response obviously appears strongly smoothed as can be seen also in the polar-plot.

For 5/90 deg: The last row displays the extreme case, where the original function is provided only on two scan-lines. Interestingly the balloon contours still display some of the main features of the directivity of a rectangular piston. However, as already stated, the level in the extrapolated regions turn out to be too high. This is understandable because the original function provides interference whereas the IDW does not.

Compared to the Spherical Harmonic approach the IDW seems more stable and easy to adjust. During experiments the Spherical Harmonic approach often failed, needed special regularization and asked for a careful adjustment of the number of modes. The IDW on the other hand always yields a meaningful result, however, often blurred and too optimistic.

3. MEASUREMENT

The measurement of a loudspeaker 3D directivity, as described in [1], requires the loudspeaker impulse response to be sampled with a 5 degree resolution "equi-angular" spherical pattern. This means a total of 2664 measurement points [7].

Among the different approaches to acquire the 3D measurement set we may: use multiple microphones, move one or more microphones around the loudspeaker, use a single fixed microphone coupled with

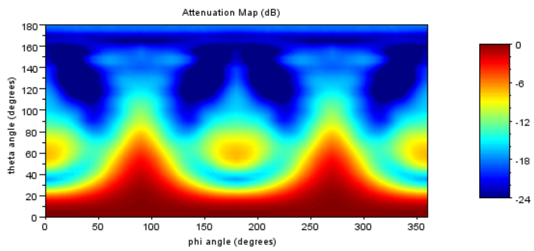

Fig. 5: Color-map plot: directivity balloon is projected on a plane.

a mechanical system to orient the loudspeaker towards a given direction or any mix of the above. It is outside the scope of this paper to enter into the details of 3D measurements. We would like to point out here the complexity of such kind of setup, both in terms of needed hardware and time.

A simpler and more cost-effective alternative is to sample the sphere over only a few scan-lines. Using a single fixed microphone and a single turntable (or equivalent) it is easily possible to sample the loudspeaker response over the horizontal and vertical scan-lines. This can be done by placing the loudspeaker in vertical and horizontal position on the turntable, the rotation of the turntable corresponds to an apparent rotation of the microphone around the loudspeaker.

In this case the full balloon response has to be extrapolated from the available data. We will compare here the results of 5 degrees full-balloon measurements with an extrapolation made with the IDW method.

Before showing the results of real world examples, we need to introduce another graphical representation of the 3D directivity in the form of a color-map as rectangular projection of the sphere: the two spherical coordinates angles φ and θ are projected on the x- and y- axes, while the sound pressure level is represented by a color shade (see figure 5). Despite the distortion introduced by such projection, this allows for a simple representation of the difference between 5 degrees 3D balloon measurement and data extrapolation made with the IDW method.

We will report here two different case studies: a 1 inch compression driver on a 90x60 exponential horn, and a small bookshelf 2-way sealed loudspeaker box. Throughout these examples $u = 3$ has been used for the IDW extrapolation and data has been smoothed by third octave bands.

3.1. Compression driver on exponential horn

The driver-horn assembly has been measured in 3D using two computer controlled turntables, with an angular resolution of 5 degrees. Since the source has a symmetry along the vertical and horizontal planes, only a quarter of a sphere has been sampled and data has been then mirrored.

Figure 6 (left-top) shows the measured balloon plot (in this plot the surface is distorted in such a way that the radius of the sphere is proportional to the sound pressure level) of the driver at 8 kHz frequency band, where the effect of the rectangular aperture is clearly visible. In the same figure 6 (right-top) the balloon extrapolated from horizontal and vertical scan-lines using the IDW method is shown.

The absolute error in dB between measured and data extrapolated is shown in the bottom row of figure 6 as a color-map plot.

While the absolute error is not small, the overall balloon shape is not affected, as previously found with the sincsinc test function.

3.2. Small 2-way loudspeaker box

The 2-way box has been measured in 3D using the same setup as above. Due to the horizontal symmetry only half sphere has been sampled and then data mirrored.

This loudspeaker exhibits an interference pattern between the two drivers at the crossover frequency. The interference can be seen in the 3.15 kHz one third frequency band measured balloon (figure 7 (left-top)).

Figure 7 (mid-top) illustrates the balloon extrapolated from horizontal and vertical scan-lines using the IDW method. A color-map plot of the absolute error of the IDW extrapolation using the above scan lines is in figure 7 (left-bottom). Here again, while the balloon presents quite a complex behavior featuring multiple side-lobes, the overall shape is kept and side-lobes are present, albeit attenuated.

The error of the IDW extrapolation can be reduced using more scan-lines. We will propose here a very

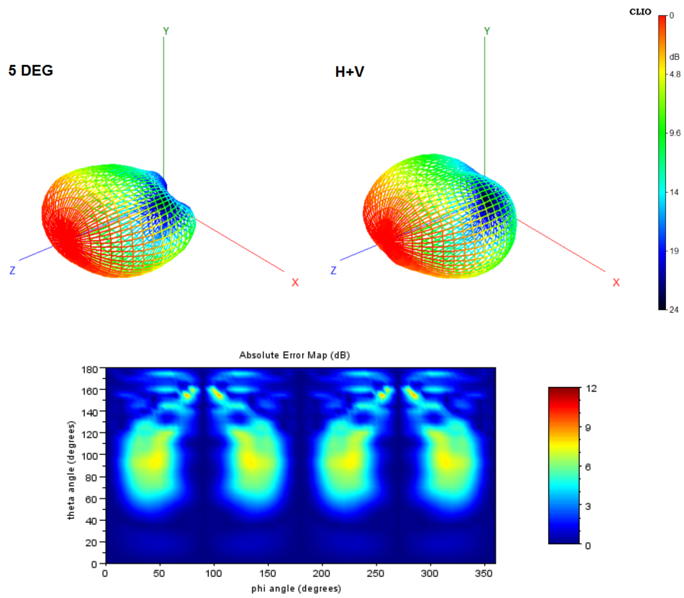

Fig. 6: Driver on exponential horn balloon at 8 kHz: (left-top) 5 degree measured, (right-top) IDW extrapolation, (bottom) absolute error.

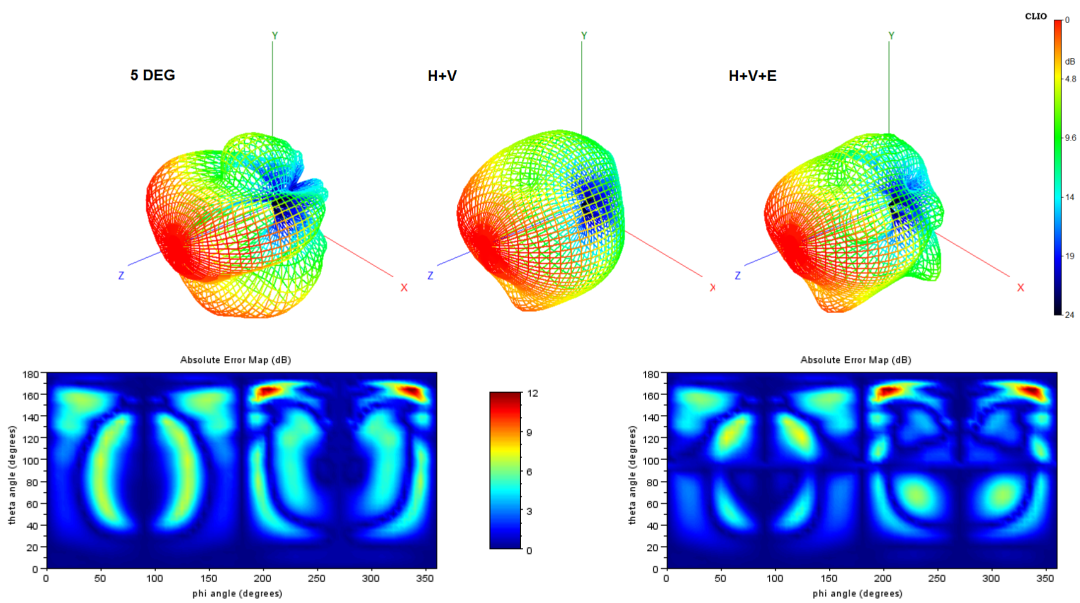

Fig. 7: 2-Way loudspeaker at 3.15 kHz: (left-top) 5 degree measured, (mid-top) IDW extrapolation H+V, (right-top) IDW extrapolation H+V+E, (left-bottom) absolute error H+V, (right-bottom) absolute error H+V+E.

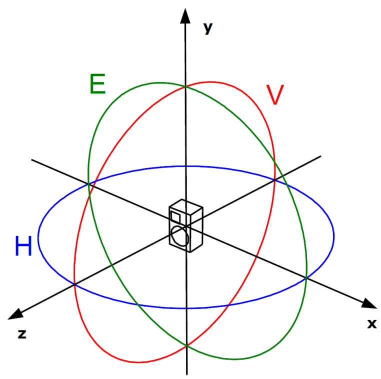

Fig. 8: Horizontal, Vertical and Equatorial Scan-Lines.

simple setup to collect an additional scan-line that, in an analogy with earth science, we call “equatorial” (figure 8). The proposed setup uses a single turntable and a single fixed microphone, as in the case of horizontal and vertical scan-lines. In order to collect the equatorial scan-line, the loudspeaker under test has to be placed on the turntable facing up (pointing towards the ceiling).

The balloon plot of the data extrapolated with IDW method using horizontal, vertical and equatorial scan-line is presented in figure 7 (right-top) and the error is shown as color-map plot in the same figure (right-bottom). The presence of the “equatorial” data helps to reduce the error, while the required additional measurement effort is minimum.

4. CONCLUSIONS

This paper reports on the investigation of the application of the Inverse Distance Weighting method for extrapolating incomplete 3D directivity data.

The IDW is not a substitute for the complete dataset as can clearly been seen by the investigation of the canonical SincSinc function and by the error-maps of measurements done. However, the IDW can provide rough estimates for the unknown areas even under the extreme case, where only a horizontal and vertical scan-line is available. In any case the IDW provides bounded results and is easy to apply.

As most measurement setups go along scan-lines it seems beneficial in this context to measure the device under test also equatorial.

Future research will investigate refinements of the IDW and will look into other possibilities of extrapolating the balloon-plot.

5. AKNOWLEDGEMENTS

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